Technomics
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Bridging the Gap: Leveraging Micro Agile Data in Macro Planning Estimates

Joint IT/Software Cost Forum

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Abstract

A fundamental gap persists in Agile software implementation. At a micro level, we are awash in data, inundated with stories and story points from a life cycle management tool like Jira, Redmine, or VersionOne. At a macro level, we struggle to adequately define functional requirements sufficient to support consistent sizing via function points (FP). Even if we do manage to functionally size planned future work, we often have not accrued a historical database of actual effort and cost tied directly to epics and features – the very objects we need for an apples-to-apples comparison with our program baseline. The #NoEstimates advocates throw up their hands and say that a macro-level planning estimate – five years' worth of annual budgets, for example – is futile. However, whenever we are spending "other people's money," especially the American taxpayer's, we are obliged to apply best practices in quantifying that longer-term commitment up front.

Building on previous research, this paper presents a framework for macro agile estimation based on fully analogized sizing scales that enable the application of expert judgment to produce an accurate characterization of early-stage uncertainty. It also provides a blueprint for building a database of analogies to populate such scales and presents empirical results from applying them.

https://www.dhs.gov/joint-it-and-software-cost-forum

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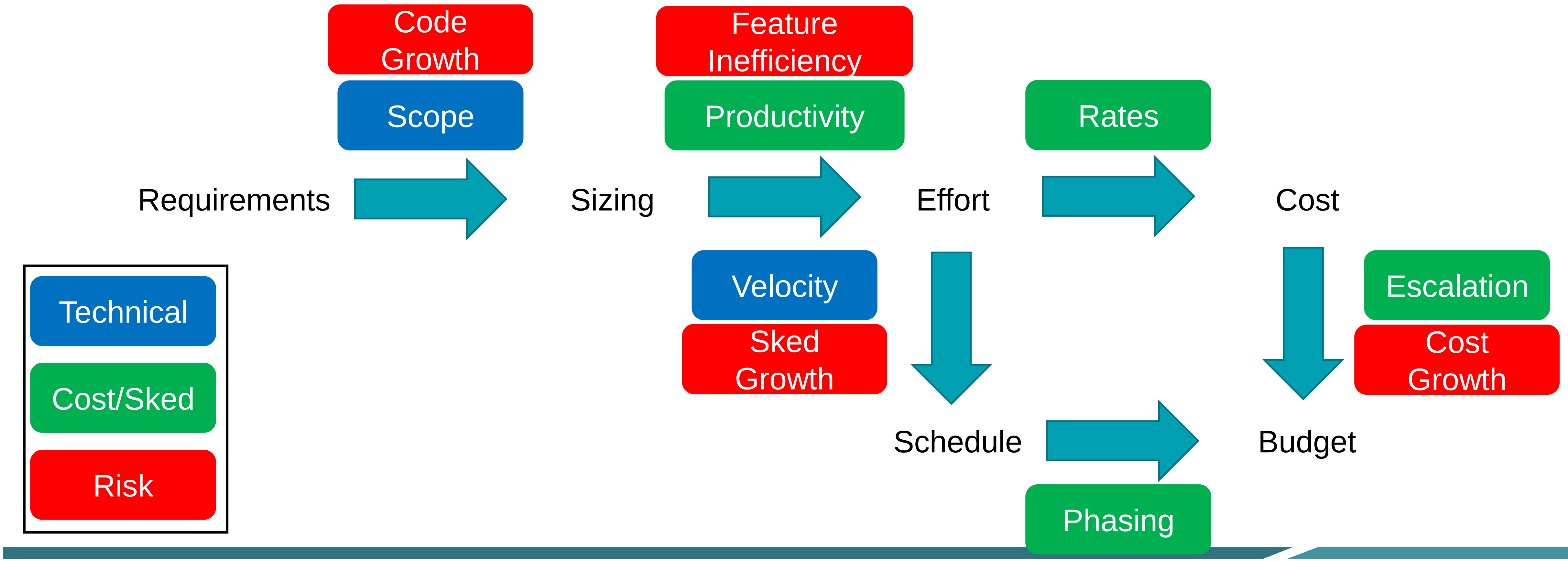
Outline

- Macro Level: The Need for Planning Estimates
- Micro Level: Agile Life-Cycle Management Data
- The Gap: Disconnects Between Macro and Micro
- Building Bridges: Approaches to Spanning the Chasm
- Testing the Pillars: Agile Infrastructure
- Next Steps: Data and Training

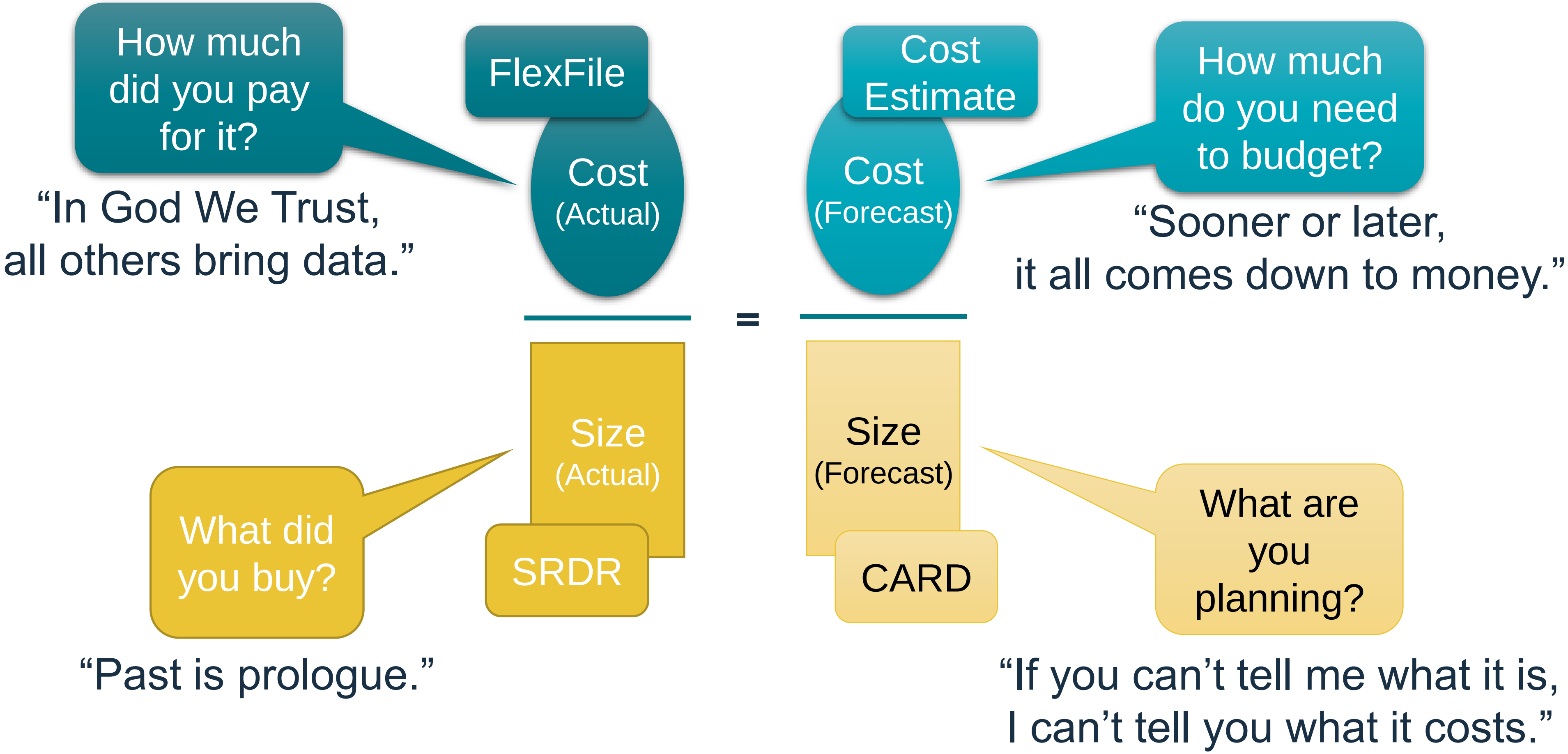
Software Estimating Data Flow

- In a preferred detailed Software Cost Estimating / Inputs Risk scenario, each component is modeled separately, with data-driven uncertainty



Cost Analysis in One Picture

- There are three key ingredients to Cost Analysis



Macro Planning Estimates

- Once primed, the Agile Software Factory churns out code within a Time Box of Sprints
  - Cost (scrum team size) + Schedule (Program Increments) → Performance (new capability)
- However, there is an early (and ongoing) need for Planning estimates to determine Resource needs and expected deliveries
  - Performance → Cost + Schedule
- Neither the Software Pathway nor DevSecOps obviate the need for these Planning estimates
  - Our goal is to connect them to Micro level Agile data

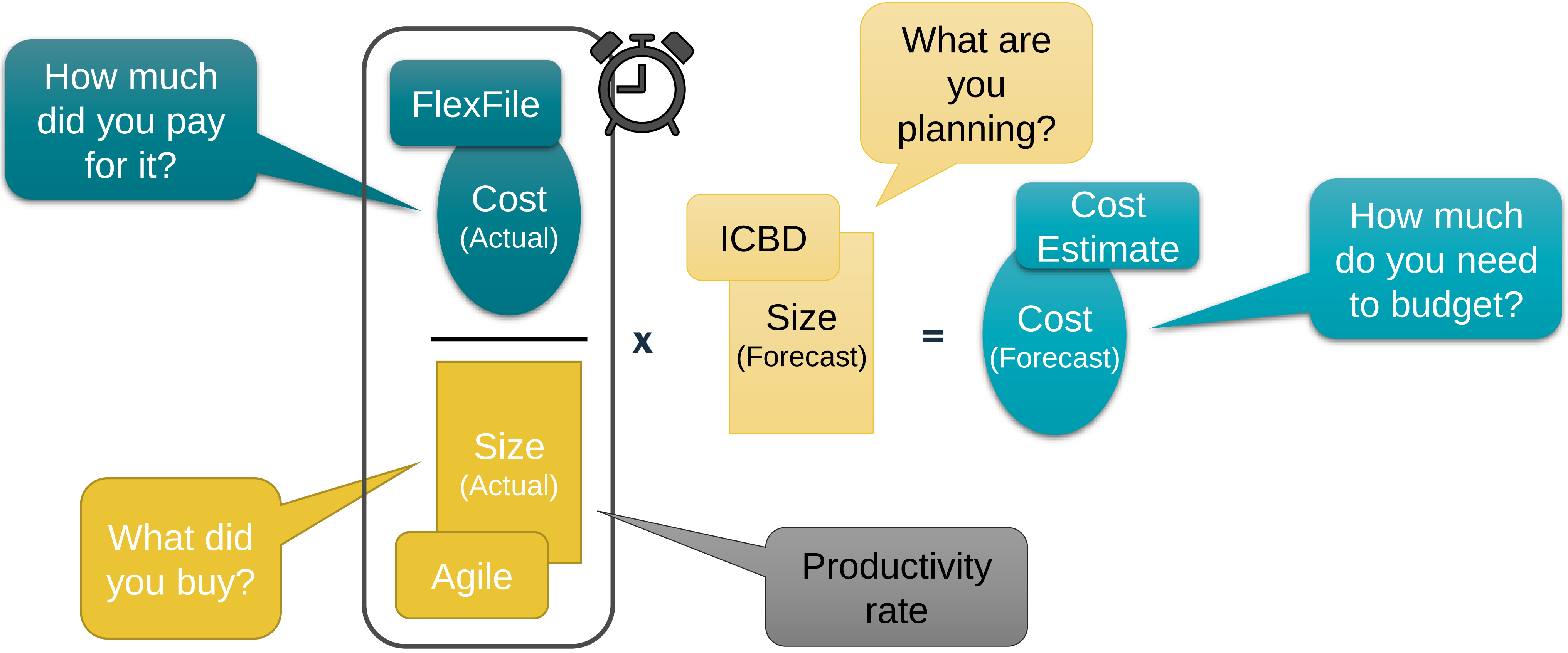
Inverting – but not subverting – the Iron Triangle

Micro Agile Data

- Metadata:
  - Sprint, epic
- Planning data:
  - Story, scrum team, story points
- Actuals:
  - Hours

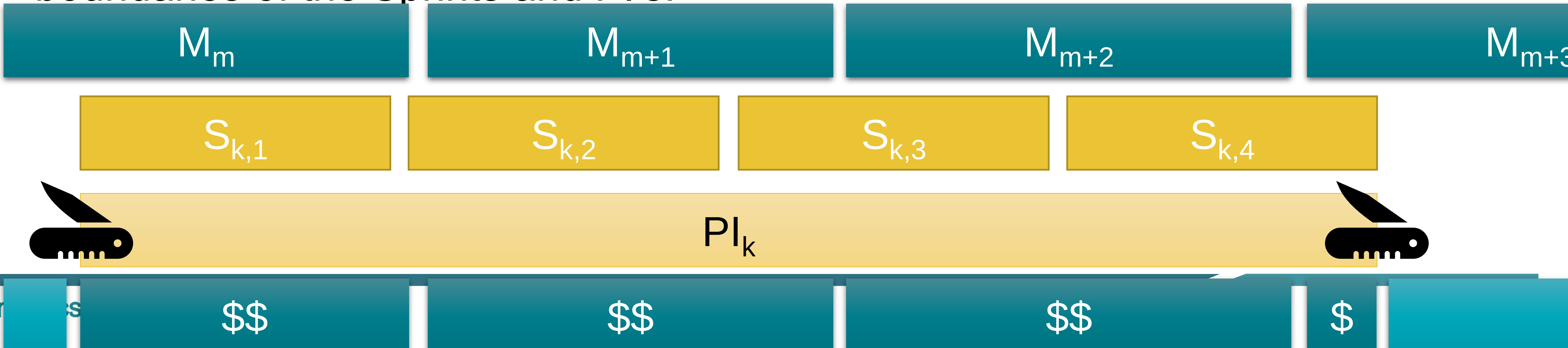
Agile Productivity Metrics

- The "*Hours per LOC* thought process" applies actual Productivity
- Time frames may not agree (e.g., Sprints vs. accounting months)



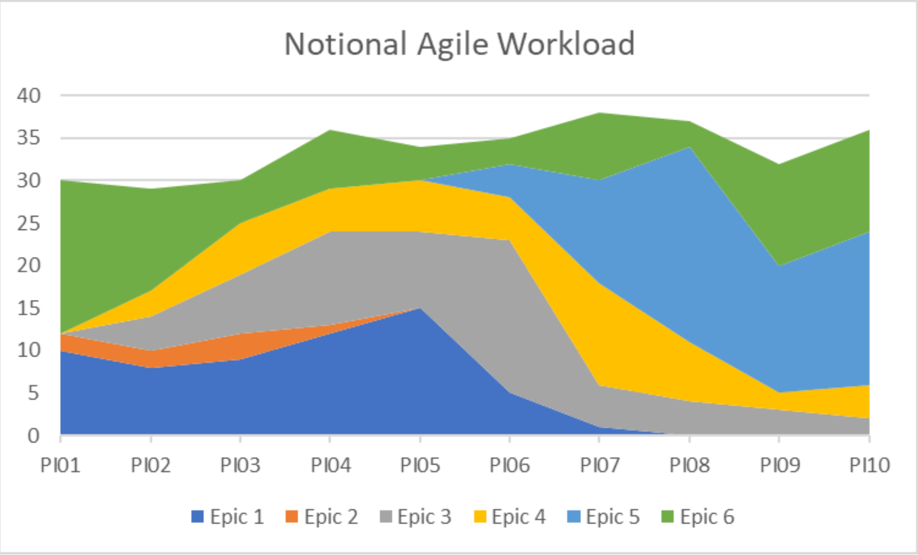
Aligning Time Frames – The "Wiener Slicer"

- Cost and Hours data: Accounting months
  - '4-4-5' Accounting calendar
- Agile data: Quarterly Program Increments (PI's)
  - Comprising four three-week sprints
- Preferable to map Cost and Hours data to Agile timeframes
  - Dollars more "fungible" than Features…
  - Imagine that each Accounting month is a hot dog, sliced by the razor-sharp boundaries of the Sprints and PI's!



Longitudinal Agile Dev – The "12-Lane Highway"

- Scaled Agile Development enables progress on several Epics concurrently
  - Deploying technology when appropriate … good procrastination?
- Downside is that parallel development continues for years at a time
  - "This is not a Gantt chart … it's a 12-lane highway!"
- The challenge is that data have to be extracted longitudinally
  - Requires adequate tagging in LCM tool *and* Accounting system



"When everything's a priority, nothing's a priority!"

Planning Poker and Fibonacci Numbers

- Alternate sizing method is Planning Poker
  - Commonly uses Fibonacci numbers for sizing via Story Points
  - In some alternative formulations, larger sizes are replaced with “rounder” numbers
  - Often visualized using fruits!
- Combines “additive” and “multiplicative” features:
  - Sum of any two consecutive sizes is equal to the next largest size
  - Ratio of consecutive sizes *approaches* a constant
- Fibonacci numbers are the sequence starting with 1 and 1, and whose subsequent entries are the sum of the two previous numbers
  - 2 = 1+1, 3 = 1+2, 5 = 2+3, 8 = 3+5, 13 = 5+8, 21 = 8+13, 34 = 13+21, etc.

Fibonacci Numbers and the Golden Ratio

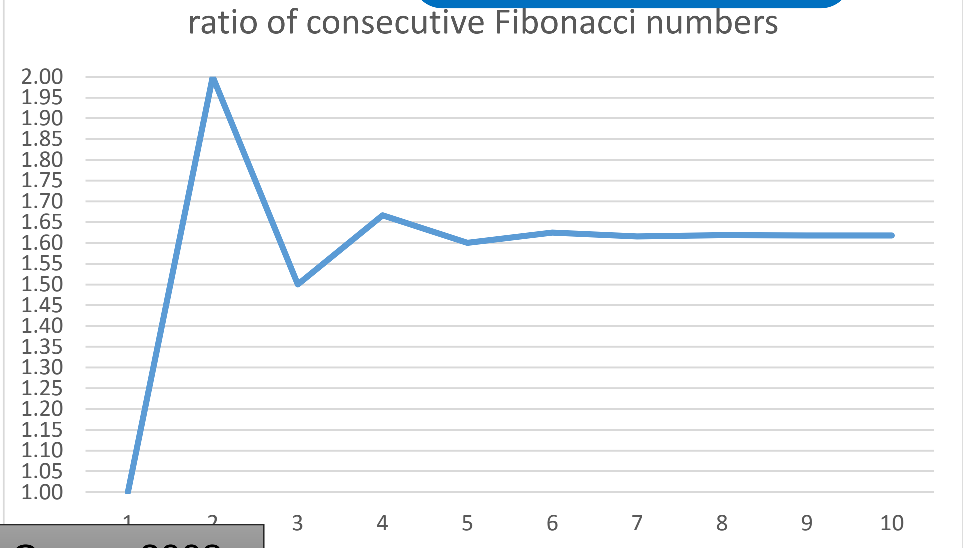
- Because the Fibonacci sequence is additive, the ratio between consecutive terms is *not* constant
- However, the ratio does quickly converge to a constant
  - It turns out that this is the Golden Ratio!

$$\phi = \frac{1+\sqrt{5}}{2} = 1.618\ldots$$

$$F_n = \frac{1}{\sqrt{5}}\left[\phi^n - (1-\phi)^n\right]$$

n	Fn	closed form	ratio	low/high
1	1	1		
2	1	1	1.000000	low
3	2	2	2.000000	high
4	3	3	1.500000	low
5	5	5	1.666667	high
6	8	8	1.600000	low
7	13	13	1.625000	high
8	21	21	1.615385	low
9	34	[illegible]	[illegible]	high
10	55	[illegible]	[illegible]	low
11	89	[illegible]	[illegible]	high
12	144	[illegible]	[illegible]	low
[illegible]	832,040	832,040	1.618034	low

Factor = 1.618:1
Range = 144:1



The Golden Ratio: The Story of PHI, the World's Most Astonishing Number, Mario Livio, Crown, 2008.

Micro Agile Plans vs. Actuals

- Stories are typically assessed in Story Points on a Fibonacci or T-shirt scale
  - They are not re-assessed *ex post facto*
- If scrum team assessment does not include hours directly, an “hours-per” factor must be assumed for purposes of comparing Plans and Actuals
  - 8 hours/Story Point is a good default starting point
  - Overall distribution is robust to choice of factor

Sizing Approaches – Definitions

both

T-Shirt Sizing: Popularized by Agile Teams (S/M/L/XL)

micro

Planning Poker: Gamified technique to gather input from group

Fibonacci Numbers: "borrowed from nature … allows relative sizing"

Story Points: capture complexity, breadth, and risk

macro

Function Points (FP): based on logical data groups and processes

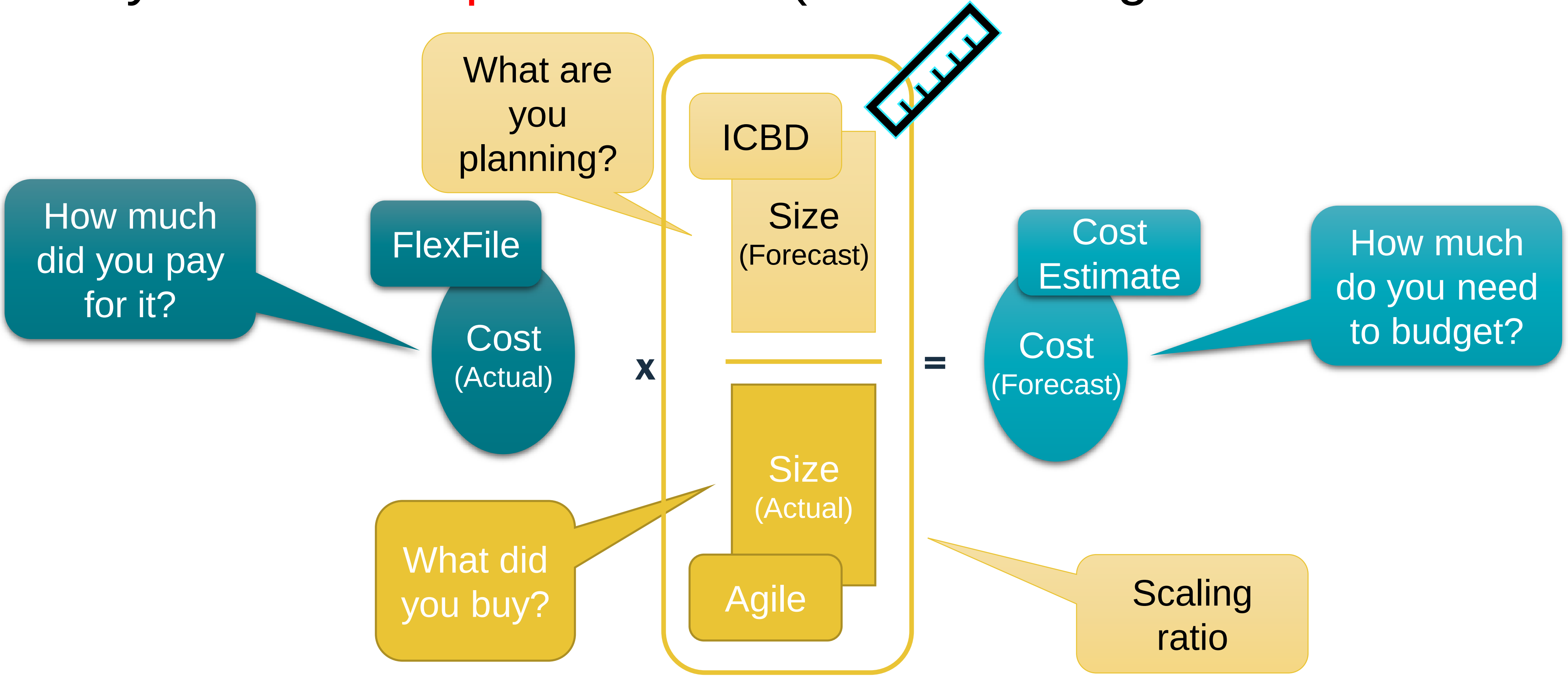
Simple Function Points (SiFP): three transactional processes

Source Lines of Code (SLOC): quantitative measurement

"an indication of effort"

Macro-Level vs. Micro-Level

- The "*Analogy* thought process" scales actual cost by a Size ratio
- Sizes may be in incompatible units (if not downright incommensurable)



Different Kinds of Bridges

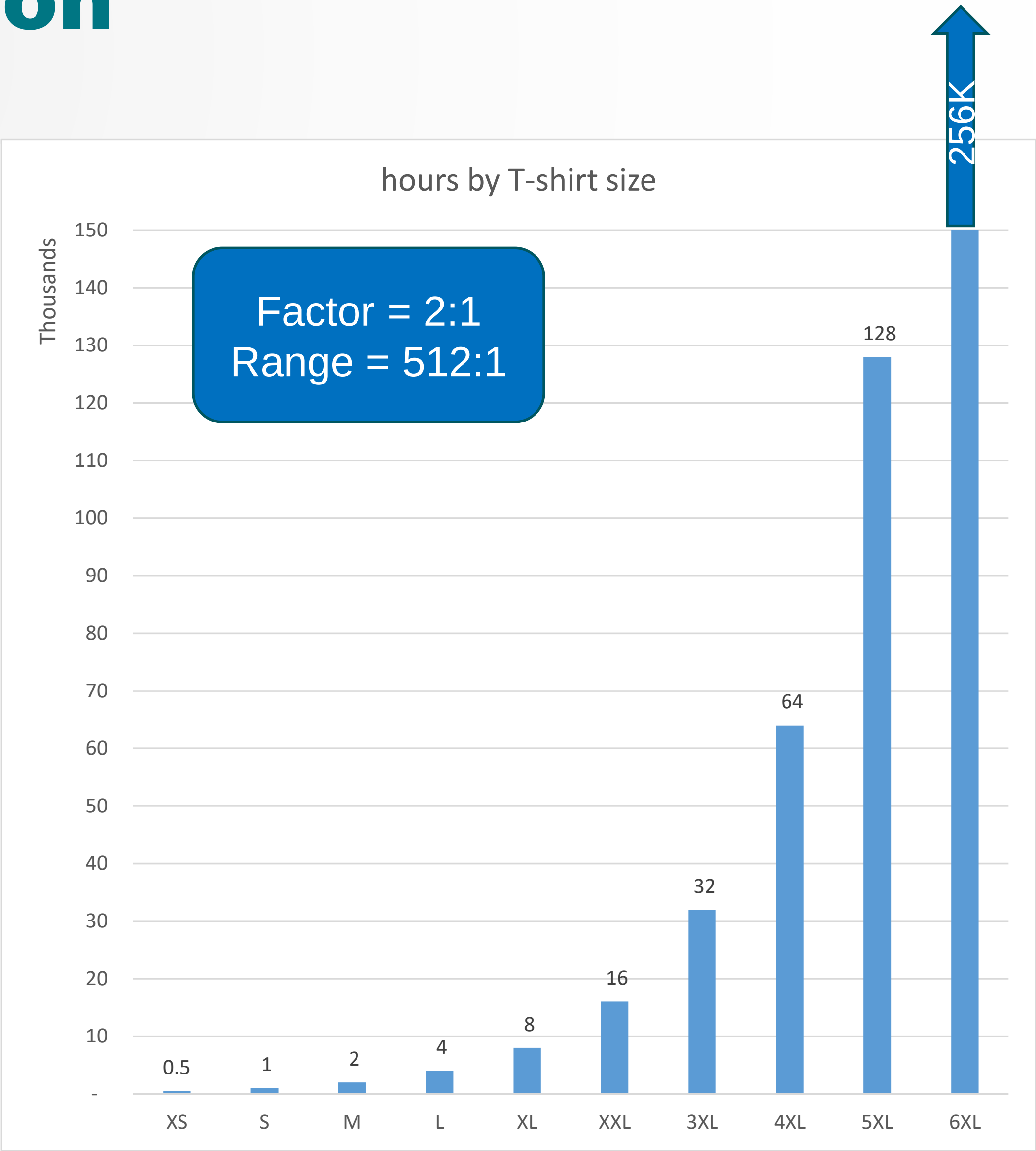
- Rope Bridge: T-shirt sizing
  - Only a tenuous connection with Micro data (implicit SME experience)
- One-Lane Covered Bridge: SLOC-based
  - Treat all new capability as SW Sustainment (Perfective/Adaptive) – "software is never done!"
- Cantilever Bridge: Function Points (FP)
  - If sufficient requirements detail is available, manual and/or automated FP methods can be used to capture functional size[1]
- Suspension Bridge:
  - Fully-analogized T-shirt size scales – ideal blend of Macro actuals and Expert Judgment

Salting the bird's tail

1. NLP for Functional Sizing, David H. Brown, et al., JITSWCF, 2022.

T-Shirt Sizing Risk – Introduction

- T-Shirt Sizing is purposefully an exponential scale (aka logarithmic)
  - Similar to the use of Fibonacci numbers and “planning poker” in Agile
  - Other common logarithmic scales include Richter (earthquakes) and Decibel (sound)
- Going-in Risk position is that SME assessments could very easily be off by one T-shirt size in either direction
- Straightforward math leads to growth percentages and CVs under various distributional assumptions



T-Shirt Sizing Risk – General Framework

- Premise: A variation of the "double-or-half" thought experiment establishes a specific probability distribution
- Risk: Compute the **mean** of the probability distribution
  - Compare to the original point estimate (*H* hours) to establish a Cost Growth Factor (CGF), and equivalent **percent growth** (on average)
- Uncertainty: Compute the **variance** of the probability distribution
  - Compare standard deviation to the original point estimate ("pseudo CV") and estimate with growth to determine Coefficient of Variation (**CV**)
- Refinements:
  1. From discrete to *continuous* outcomes
  2. Incorporating degree of *confidence*
  3. Adjusting *beyond* "double-or-half" based on confidence
  4. Generalizing to ratios other than two

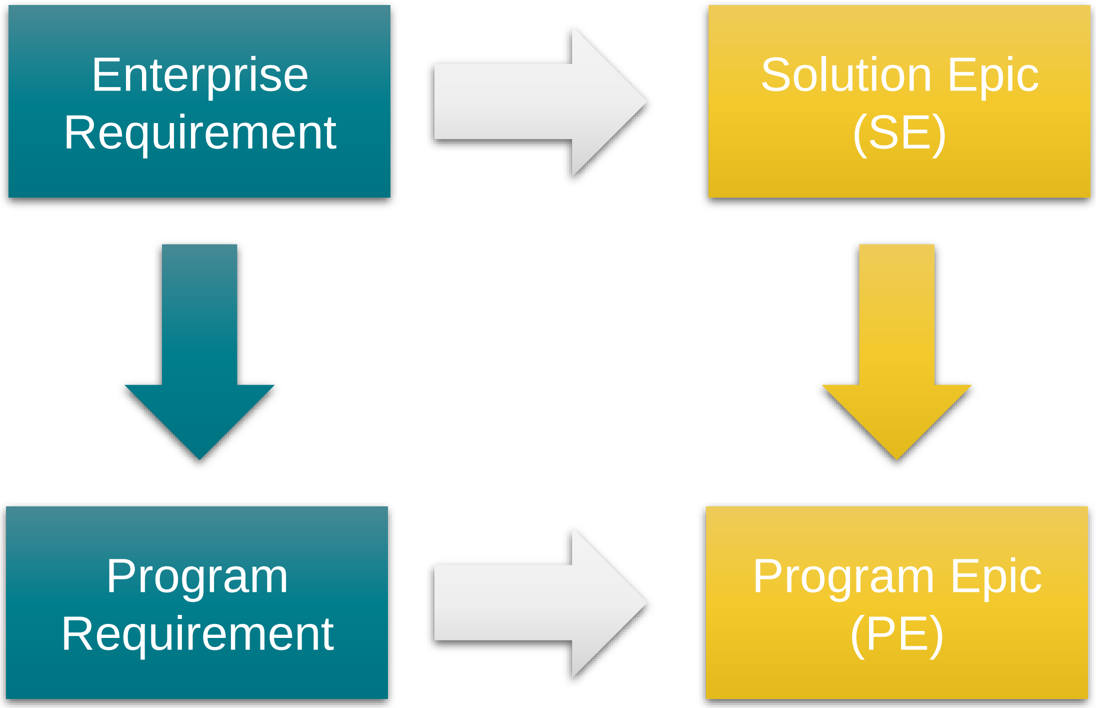
Architecture, Reqts, Design – The "Layer Cake"

- Enterprise Release Management (ERM) specifies hierarchy

Architecture	Requirements	Design
	Strategic	
	Enterprise	
	Program	

Reqts and Design – The "City Block Problem"

- Verification and Validation of Requirements ideally occurs at multiple levels
- Decomposition of Requirements in the Reqts Db
  - Decomposition of Epics in the LCM tool
  - These are both "Avenues"
- Traceability from Reqts to Design along the "Streets"



Which is the better way to "go around the block"?

Actuals Trace – The "Broken Ladder"

- Verification and Validation of Requirements ideally occurs at multiple levels
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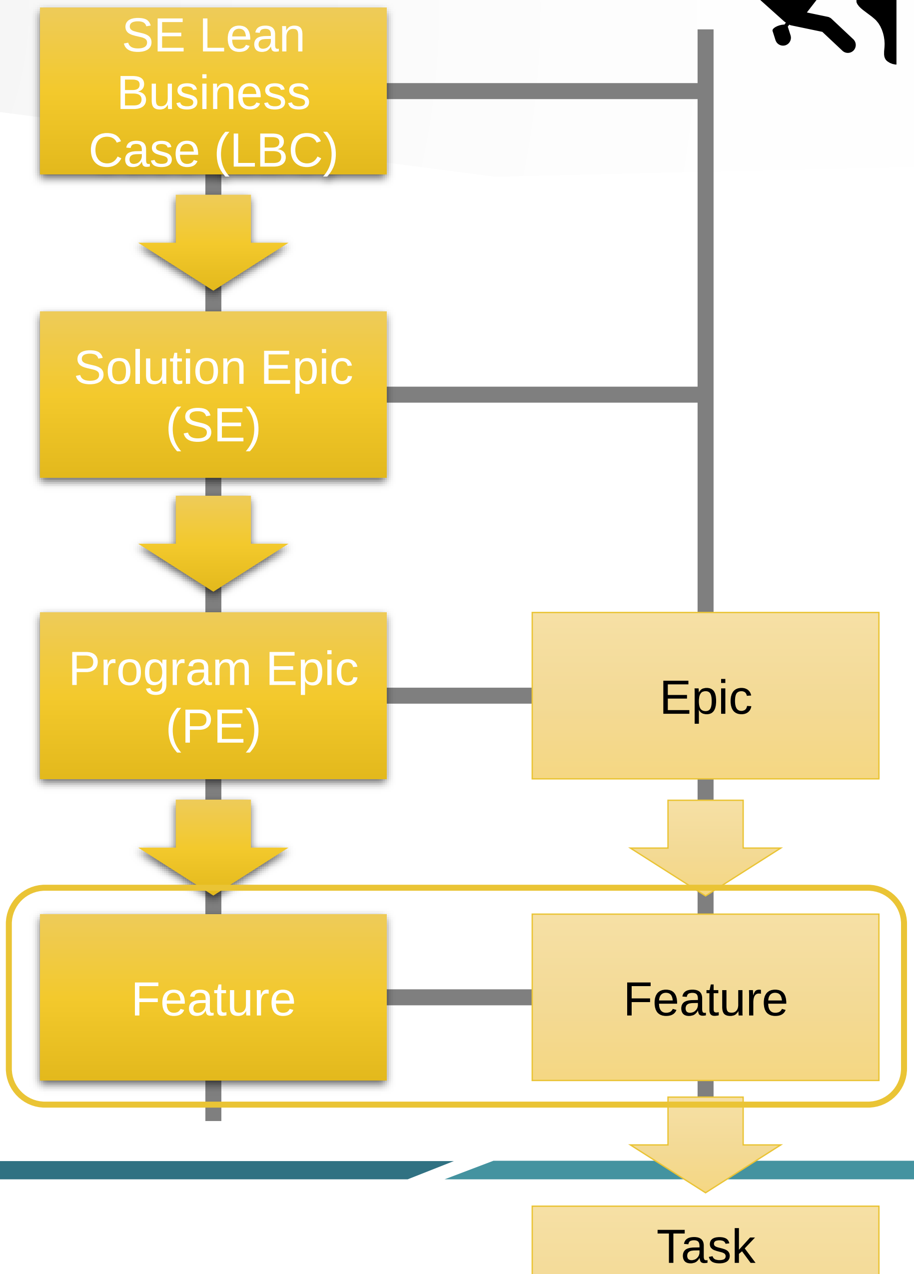
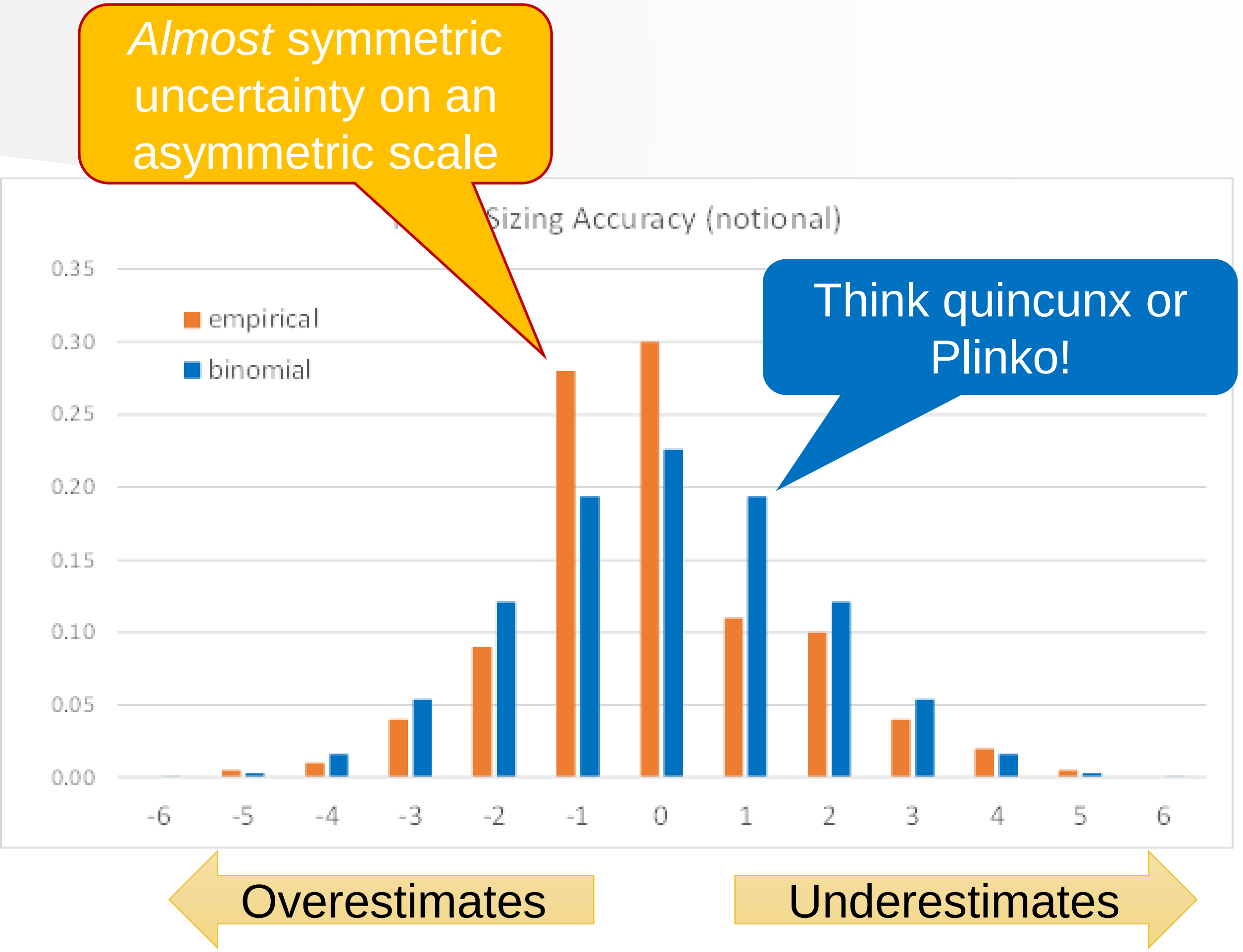


Which is the better way to "go around the block"?

Micro-Sizing Accuracy

- As presented, T-shirt sizing is Macro level, whereas Fibonacci numbers are Micro level
- Still gathering empirical evidence on Macro-sizing accuracy
  - Initial evidence for Micro-sizing is largely consistent with hypothesized model
  - Except there may be *many* coin flips, not just one…



Self-Similar Scales and the Ideal Ratio

- Self-similar scales are fractal in that misestimation will result in growth (or reduction) by the same ratio regardless of position on the scale
- Candidate ratios (R):
  - Two (2.0) – T-shirt Sizing
  - Phi (1.618…) – Planning Poker (Fibonacci numbers)
  - e (2.718…) – base of the exponential function that is its own derivative!
- It is proposed that these approximately bound the reasonable set of choices
- Related issue is “top-down” vs. “bottom-up”
  - Size more complex pieces of work as whole (initially) or force decomposition

Empirical Testing of Scales

- Approach used in previous paper on use of SME's in Cost and Risk
  - Both knowable but unknown past events (e.g., box office gross of *Avengers: Endgame*) and unknown future events (e.g., box office gross of *Thor: Love and Thunder*)
- Instead of asking for three-point estimates, ask for single best guess (closest value) from self-similar scale
  - Does gradation of scale affect accuracy of assessments?
- Expertise in subject area vs. expertise in uncertainty assessments

"Teaching Pigs to Sing: Improving Fidelity of Assessments from Subject Matter Experts (SMEs)," Peter Braxton and Richard Coleman, ICEAA Washington Chapter, June, 2012.

Expert Judgment vs. Expert Opinion

- Expert Opinion = estimate is presented as a direct assessment by SME with no apparent basis
- Expert Judgment = SME uses or interprets data as the basis of the estimate, or at worst makes a direct assessment as to the scope on which the estimate is based (e.g., software sizing!)
- It is hypothesized that sizing and similar assessments can be improved by labeling each notch on the scale with an actual example reflecting that approximate size
  - Transcends Expert Opinion with a sort of a "stealth" Analogy
  - Heights of mountains, e.g., could be used in empirical assessment

Cost Estimating Body of Knowledge (CEBoK®), Module 2 "Cost Estimating Techniques," ICEAA, 2013.

From Single-Point Analogy to Analogized Scales

- Benefits of an explicit Basis and Rationale:
  - *Independently verified* before the fact
  - *Empirically measured* after the fact
- "Analogizing" the self-similar scale
  - Augment or replace numerical values with historical examples
  - Similar to Mohs scale (mineral hardness), Beaufort scale (wind)
- Double "stealth"
  - Analogy estimate masquerading as Expert Opinion/Judgment
  - Three-point estimate masquerading as one-point estimate

Experimental Formulation

- Six basic treatments
  - Scale labeling: numbers only, analogies only, or both
  - Scale ratio: 1.5 or 2.0
- Experiment #1: Heights of Mountains
  - Unknown but knowable, generally relatable

scale (ft)	mountain	location	elevation (ft)
500	Driskill Mountain	Louisiana	535
1,000	Woodall Mountain	Mississippi	807
2,000	Mount Arvon	Michigan	1,979
4,000	Black Mountain	Kentucky	4,145
8,000	Guadelupe Peak	Texas	8,751
16,000	Mont Blanc	France	15,774
32,000	Mount Everest	Nepal	29,031

scale (ft)	mountain	location	elevation (ft)
1,000	Woodall Mountain	Mississippi	807
1,500	Crown Mountain	St. Thomas, USVI	1,555
2,250	Eagle Mountain	Minnesota	2,302
3,375	Mount Davis	Pennsylvania	3,213
5,063	Black Mesa	Oklahoma	4,975
7,594	Black Elk Peak	South Dakota	7,244
11,391	Mount Hood	Oregon	11,249
17,086	Pico Pan de Azucar	Colombia	17,060
25,629	Nanda Devi	India	25,643

Additional Experiments

- Experiment #2: Box Office Gross of Films
  - Popular films from 2010-2019 (pre-pandemic) per Box Office Mojo
  - *Not* inflation-adjusted
  - Representative of macro-level sizing
  - For a $2M to $1B range, 10-point scale (R = 2.0) or 16-point scale (R = 1.5)
- Experiment #3: Driving Distances
  - From Technomics HQ in Arlington, VA, to local and interstate destinations
  - Test the fractal nature of risk

- Experiment #2 conducted at both ICEAA Pittsburgh 2022 and ICEAA WCAC CEBoK Training

Experimental Results

	under	over
modest	3	3
significant	2	2

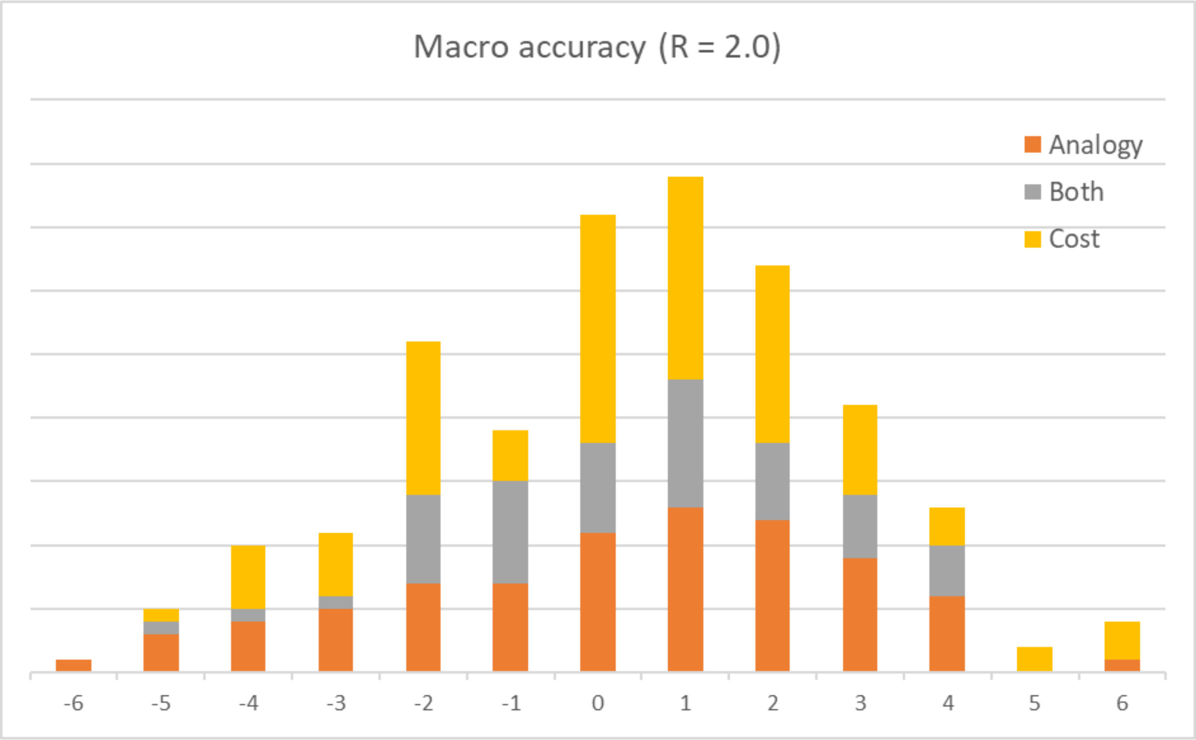
- Wisdom of the Crowds
  - While many responses were wildly incorrect, averages tend to converge to *near* the correct answer
  - Mean deviation of 0.19 (slight overestimate) across all responses
- Rule of Thirds
  - Micro level data is close to 1/3 each under, correct, over
  - Macro level data shows about 1/3, 1/6, 1/2 (i.e., greater prevalence of over)
- The Noise
  - Similar to Micro level data, being off by more than one notch is common
  - Mean absolute deviation (MAD) of 2.24 across all responses

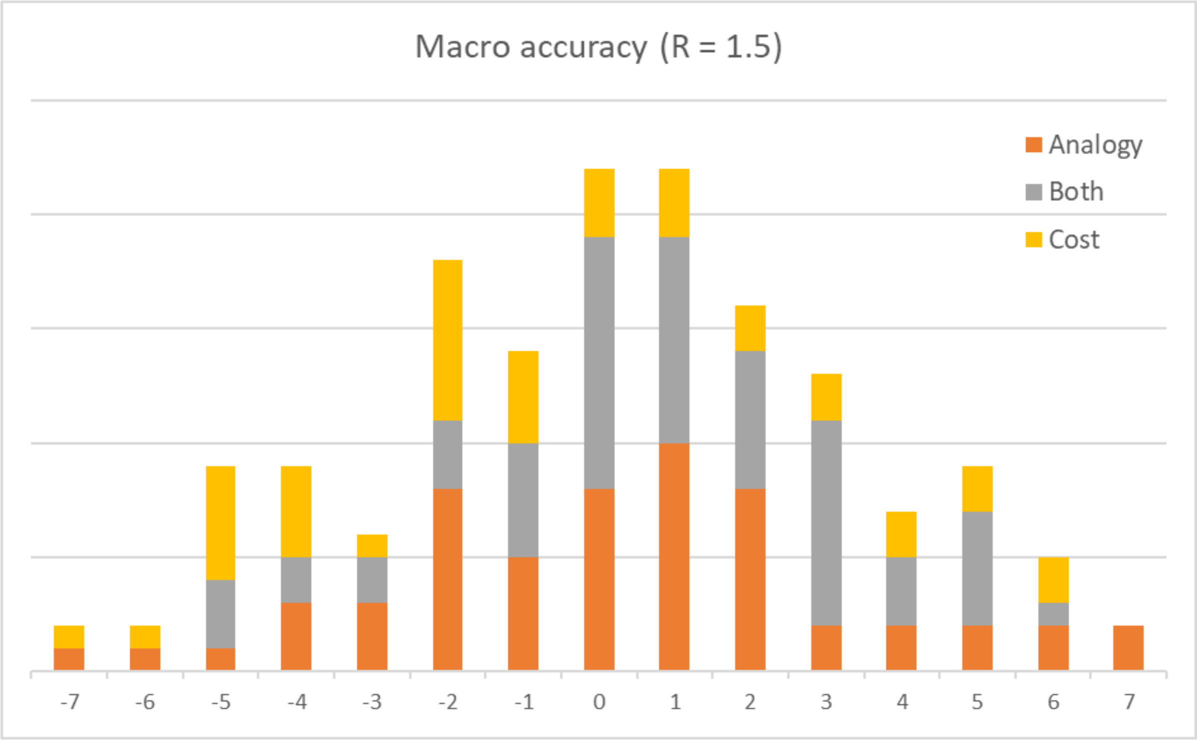
$$\frac{2.61}{1.97} = 1.324 \approx \frac{4}{3}$$

- The Signal
  - Analogy only best for accuracy, Analogy + Cost best for precision

Experimental Results Illustrated

- Histograms show six experimental treatments





Conclusions and Next Steps

- Careful treatment of Agile Micro data can make it useful to support Macro Planning estimates
  - In addition to typical ongoing PI and Sprint planning
- Persistent collection of Agile Macro data can develop a library of analogies for calibrating a self-similar scale
  - Additional training with SMEs can improve Planning assessments
- Analytical results and initial data can establish Bayesian priors
  - Adjust using ongoing assessments
- Need further research on impact of uncertainty on efficiency of capability delivery

Adapt Expert-based methods to a more data-driven approach

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Bridging the Gap: Leveraging Micro Agile Data in Macro Planning Estimates

Back-Up

Fibonacci Numbers Closed-Form Formula

- A closed-form formula can be derived, which will easily demonstrate the convergence property
- Suppose a relationship of the form

$$F_n = c \cdot a^n + d \cdot b^n$$

- Then the recursive formula will be satisfied if *a* and *b* are roots of the quadratic

$$F_n + F_{n+1} = c \cdot a^n + d \cdot b^n + c \cdot a^{n+1} + d \cdot b^{n+1}$$
$$= c(a^n + a^{n+1}) + d(b^n + b^{n+1}) = c \cdot a^{n+2} + d \cdot b^{n+2} = F_{n+2}$$
$$x^2 = x + 1 \;\rightarrow\; x^2 - x - 1 = 0 \;\rightarrow a = \frac{1+\sqrt{5}}{2} = \phi\,, b = \frac{1-\sqrt{5}}{2} = 1 - \phi$$

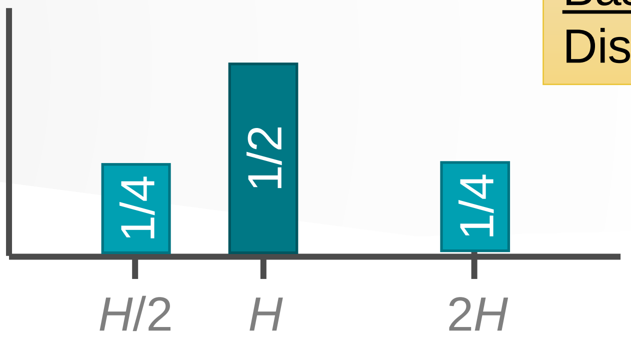
- Now we solve for the coefficients *c* and *d*

$$F_1 = 1 = \phi c + (1-\phi)d\,, F_2 = 1 = \phi^2 c + (1-\phi)^2 d$$
$$c = \frac{1}{2\phi - 1} = \frac{1}{\sqrt{5}}\,, d = \frac{1}{1-2\phi} = -\frac{1}{\sqrt{5}} \;\rightarrow\; F_n = \frac{1}{\sqrt{5}}[\phi^n - (1-\phi)^n]$$

- Since the second term vanishes as *n* increases without bound, the ratio of consecutive terms approaches *a*

Naïve Uncertainty: Coin Flips

Base Case: Discrete



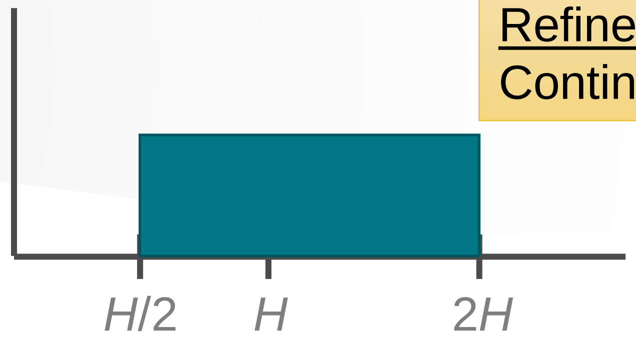
Coin flip #1: right or wrong
Coin flip #2: high or low

- Assume a Discrete distribution:
  - Most Likely = *H* hours, with a probability of 1/2
  - Max = 2*H* hours, with a probability of 1/4
  - Min = *H*/2 hours, with a probability of 1/4
- Mean is expected value: $\sum_i x_i p_i = (1/4)(H/2) + (1/2)(H) + (1/4)(2H) = \frac{9H}{8} = \left(1 + \frac{1}{8}\right) H$
  - CGF = 1.125, or **12.5%** growth over point estimate
- Variance is expected value of square less square of expected value:

$$\sum_i x_i{}^2 p_i - \left[\sum_i x_i p_i\right]^2 = (1/4)\left(H^2/4\right) + (1/2)(H^2) + (1/4)(4H^2) - \left[\frac{9H}{8}\right]^2 = \frac{25H^2}{16} - \frac{81H^2}{64} = \left[\frac{\sqrt{19}}{8} H\right]^2$$

  - CV = **48.43%**

"Maximum" Uncertainty: Uniform



Refinement #1: Continuous

- Assume a Uniform distribution:
  - Max = 2*H* hours (next largest T-shirt size)
  - Min = *H*/2 hours (next smallest T-shirt size)
- Mean is average of Min/Max: $$\frac{H/_2 + 2H}{2} = \frac{5H}{4} = \left(1 + \frac{1}{4}\right)H$$
  - CGF = 1.25, or **25.0%** growth over point estimate
- Variance is range squared / 12:

$$\frac{\left(2H - H/_2\right)^2}{12} = \frac{9H^2}{4 \cdot 12} = \left[\sqrt{3} \cdot \frac{H}{4}\right]^2 = \left[\frac{\sqrt{3}}{4}H\right]^2$$

  - CV = **34.64%**

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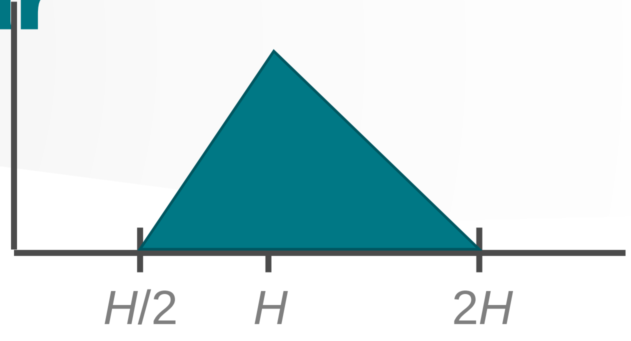
"Standard" Uncertainty: Triangular



- Assume a Triangular distribution:
  - Most Likely = *H* hours (assessed T-shirt size)
  - Max = 2*H* hours (next largest T-shirt size)
  - Min = *H*/2 hours (next smallest T-shirt size)
- Mean is average of Min/ML/Max: $\frac{H/_2 + H + 2H}{3} = \frac{7H}{6} = \left(1 + \frac{1}{6}\right)H$
  - CGF = 1.167, or **16.7%** growth over point estimate
- Variance is sum of squares less sum of pairwise products / 18:

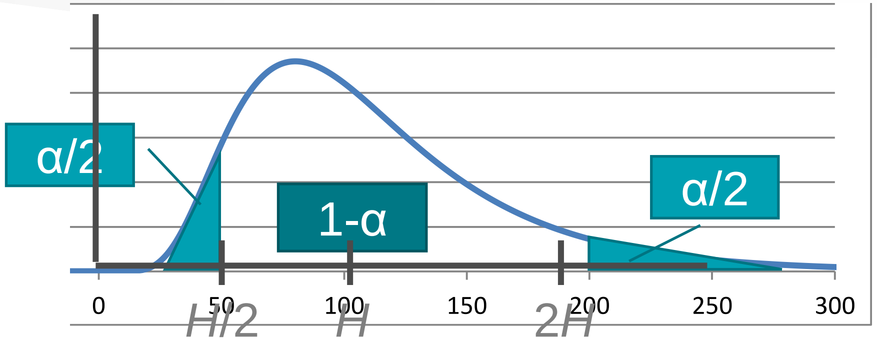
$$\frac{\left(H/_2\right)^2 + H^2 + (2H)^2 - H^2/_2 - H^2 - 2H^2}{18} = \frac{7H^2/_4}{18} = \frac{7H^2}{2 \cdot 36} = \left[\sqrt{\frac{7}{2}} \cdot \frac{H}{6}\right]^2 = \left[\frac{\sqrt{14}}{12} H\right]^2$$

  - CV = **26.73%**

"Standard" Risk: Lognormal

Refinement #3: Adjustment

- Assume a Lognormal distribution:
  - Median = *H* hours, with a probability of 1-α between *H*/2 and 2*H*
  - Right tail > 2*H* hours, with a probability of α/2
  - Left tail < *H*/2 hours, with a probability of α/2

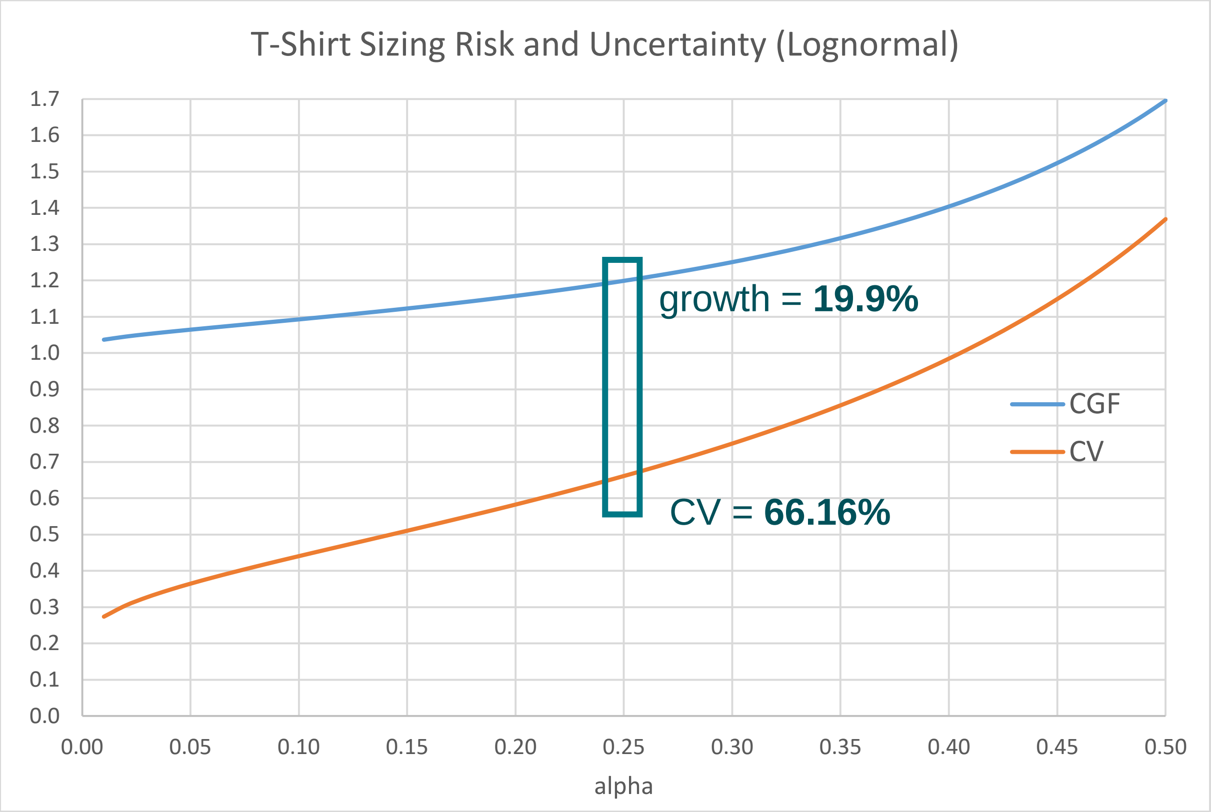


- Confidence interval of related normal is: $(lnH - ln2, lnH, lnH + ln2)$
  - So that $\Phi^{-1}\left(1 - {}^{\alpha}/_{2}\right) = \frac{ln2}{\sigma}$ $\quad \sigma = \frac{ln2}{\Phi^{-1}\left(1 - {}^{\alpha}/_{2}\right)} = \frac{1}{log_2 e^{\Phi^{-1}\left(1 - {}^{\alpha}/_{2}\right)}}$

- Mean of the lognormal is: $e^{\mu + \frac{\sigma^2}{2}}$
  - With a CGF of $e^{\frac{\sigma^2}{2}} = \sqrt{1 + CV^2}$ $\quad CV = \sqrt{e^{\sigma^2} - 1}$

T-Shirt Sizing Risk – Lognormal (Illustrated)

- Graph illustrates increase in CGF and CV as percent chance outside the “double-or-half” range increases
  - Beyond α = 0.50 (“coin flip”), values increase rapidly



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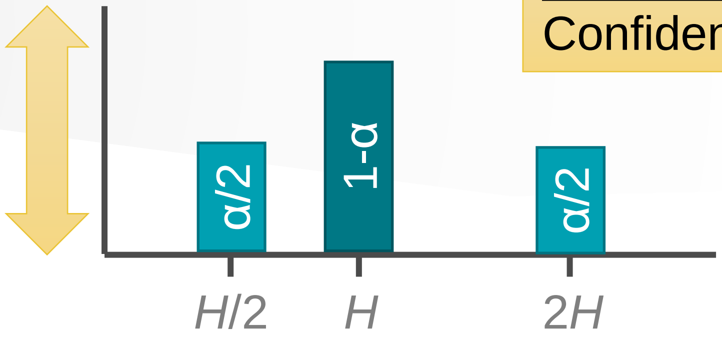
Generalization #1: Confidence

Refinement #2: Confidence



- Assume a Discrete distribution:
  - Most Likely = *H* hours, with a probability of 1-α
  - Max = 2*H* hours, with a probability of α/2
  - Min = *H*/2 hours, with a probability of α/2

In previous example, α = 1/2

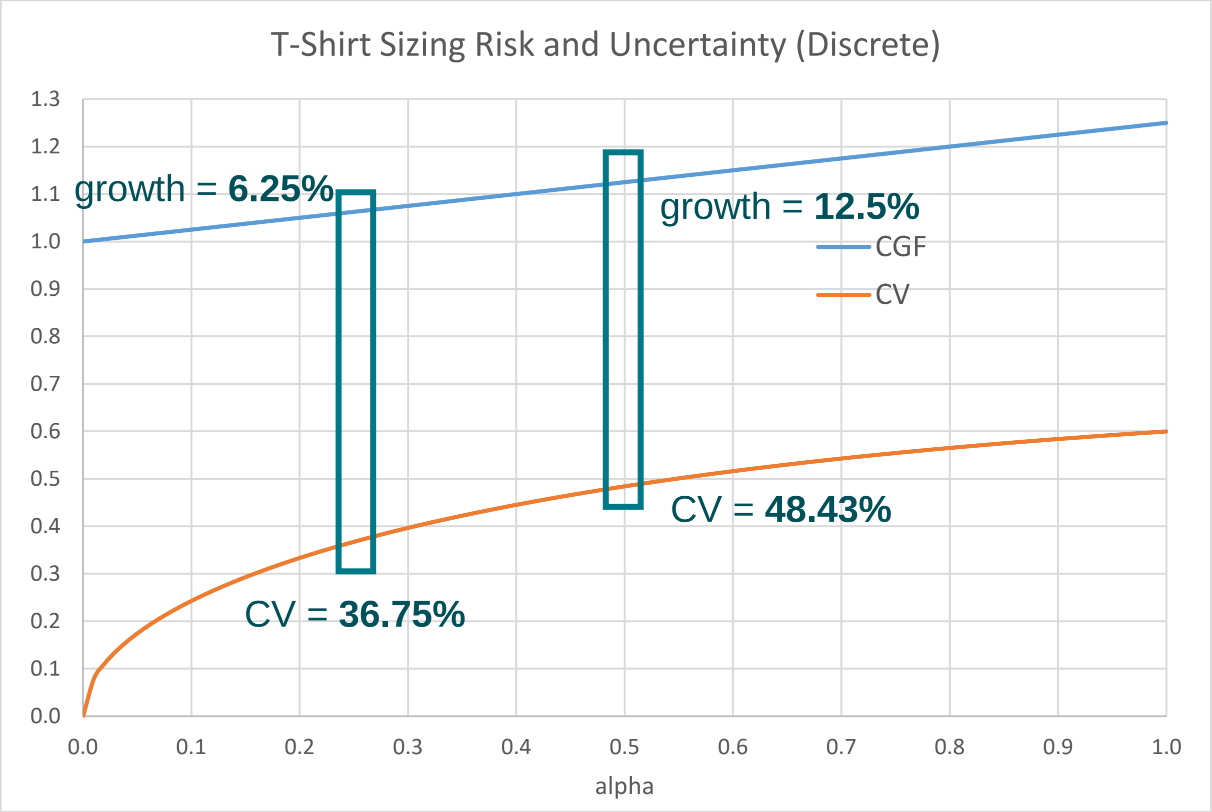
- Mean is expected value: $$\sum_i x_i p_i = \left(\alpha/_2\right)\left(H/_2\right) + (1-\alpha)(H) + \left(\alpha/_2\right)(2H) = \left(1 + \frac{\alpha}{4}\right)H$$
  - CGF = 1+(α/4), or **α/4** growth over point estimate
- Variance is expected value of square less square of expected value:

$$\sum_i x_i{}^2 p_i - \left[\sum_i x_i p_i\right]^2 = \left(\alpha/_2\right)\left(H^2/_4\right) + (1-\alpha)(H^2) + \left(\alpha/_2\right)(4H^2) - \left[\left(1+\frac{\alpha}{4}\right)H\right]^2 =$$

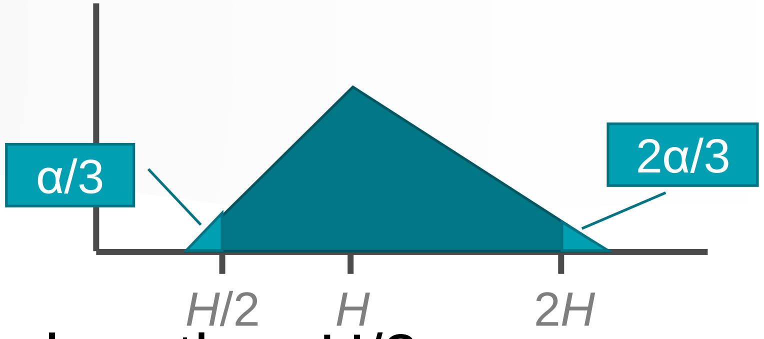
$$\left(1 + \frac{9\alpha}{8}\right)H^2 - \left(1 + \frac{\alpha}{2} + \frac{\alpha^2}{16}\right)H^2 = \frac{10\alpha - \alpha^2}{16}H^2 = \left[\frac{\sqrt{10\alpha - \alpha^2}}{4}H\right]^2 \qquad CV = \frac{\sqrt{10\alpha - \alpha^2}}{4 + \alpha}$$

T-Shirt Sizing Risk – Discrete (Illustrated)

- Graph illustrates range between always right (α=0) and always wrong (α=1), with a coin flip to determine low or high
  - Max growth is 25%
  - Max CV is 60%



Triangular Expanded – Proportional



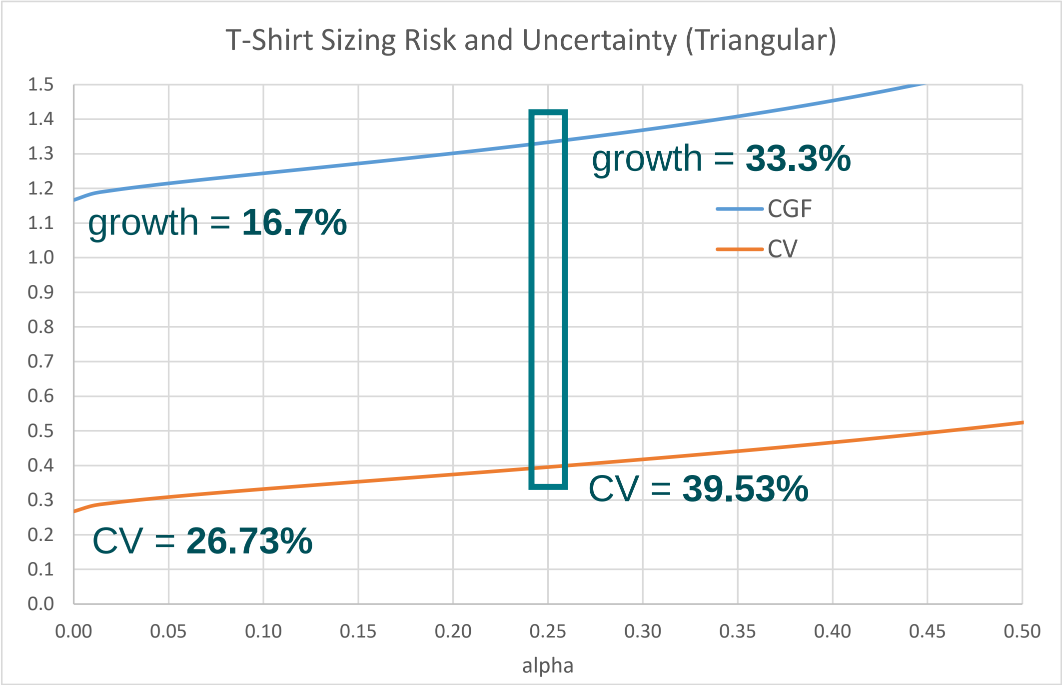
- Assume that the interval (H/2, 2H) encapsulates only (1- α) of the probability
  - That is, there is probability α of being greater than 2H or less than H/2
  - This can be split proportionally or equally
- Proportional puts $\frac{2\alpha}{3}$ above and $\frac{\alpha}{3}$ below

$$\mu = \left[\left(1 - \frac{\sqrt{\alpha}}{1-\sqrt{\alpha}}\right)\frac{H}{2} + H + \left(2 + \frac{\sqrt{\alpha}}{1-\sqrt{\alpha}}\right)H\right]\Big/3 = \left(1 + \frac{1}{6-6\sqrt{\alpha}}\right)H$$

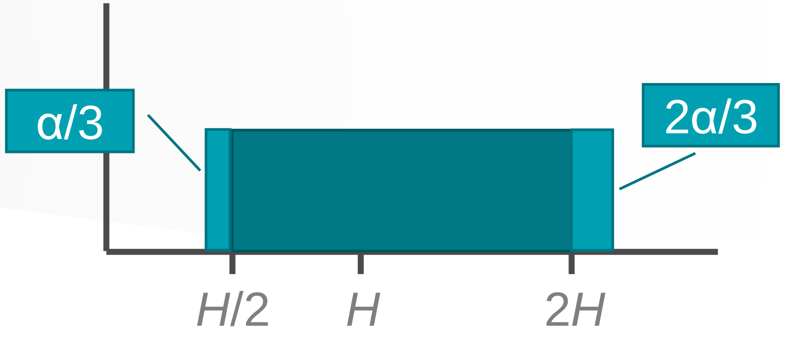
- Variance:

$$\left[\frac{\sqrt{\frac{7-4\sqrt{\alpha}}{2}}}{6-6\sqrt{\alpha}}H\right]^2$$

$$CV = \frac{\sqrt{\frac{7-4\sqrt{\alpha}}{2}}}{7-6\sqrt{\alpha}}$$



Proportional Tails – Uniform

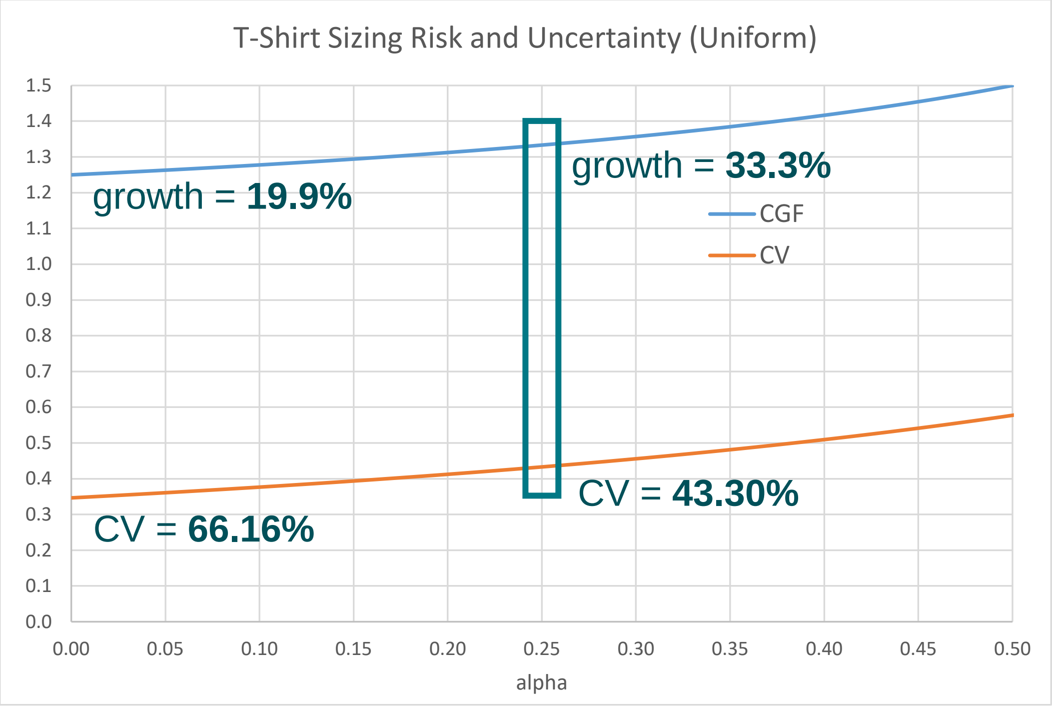


- Assume that the interval (H/2, 2H) encapsulates only (1-α) of the probability
  - That is, there is probability α of being greater than 2H or less than H/2
  - This can be split proportionally or equally
- Proportional puts $\frac{2\alpha}{3}$ above and $\frac{\alpha}{3}$ below

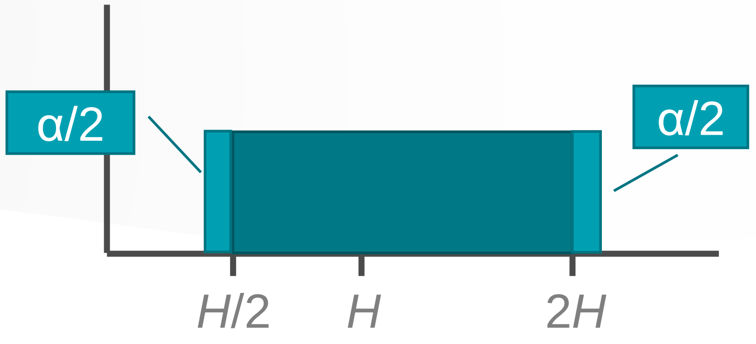
$$\mu = \left[\frac{(1-2\alpha)}{(1-\alpha)}\frac{H}{2} + \frac{(2-\alpha)}{(1-\alpha)}H\right]\Big/_2 = \frac{5-4\alpha}{4-4\alpha}H = \left(1+\frac{1}{4-4\alpha}\right)H$$

- Variance is range squared / 12:

$$\frac{(3H)^2}{12[2(1-\alpha)]^2} = \left[\frac{\sqrt{3}}{4-4\alpha}H\right]^2$$



Symmetric Tails – Uniform

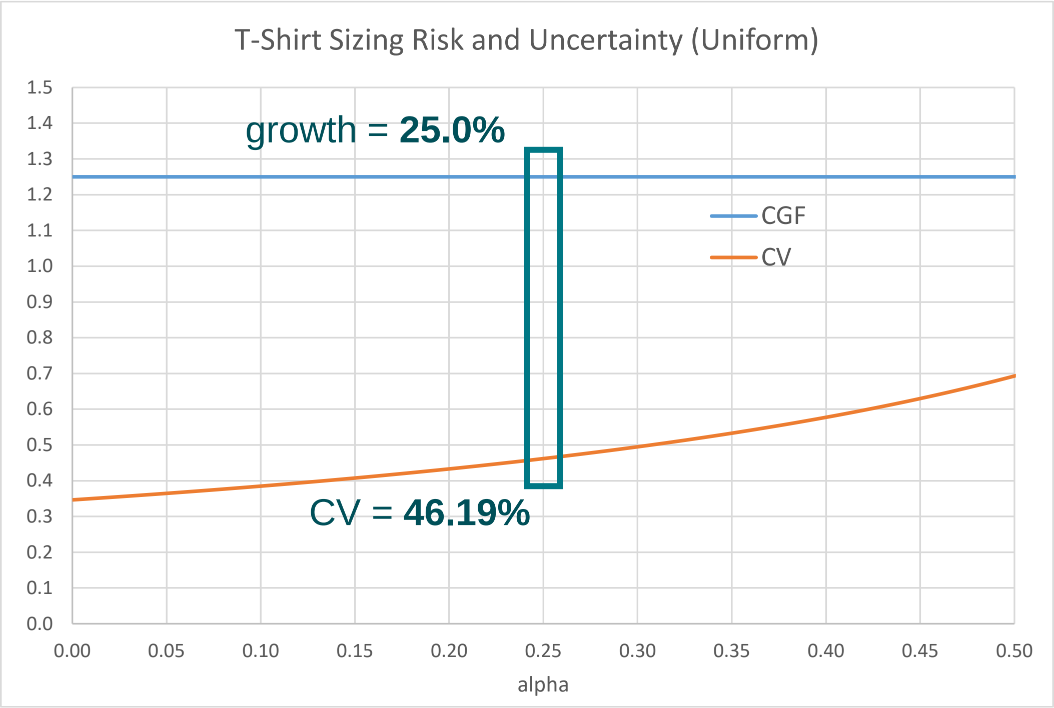


- Assume that the interval (H/2, 2H) encapsulates only (1-α) of the probability
  - That is, there is probability α of being greater than 2H or less than H/2
  - This can be split proportionally or equally
- Equal puts $\frac{\alpha}{2}$ above and $\frac{\alpha}{2}$ below

$$\mu = \left[\frac{(2-5\alpha)}{(4-4\alpha)}H + \frac{(8-5\alpha)}{(4-4\alpha)}H\right] \Big/ 2 = \frac{5}{4}H = \left(1+\frac{1}{4}\right)H$$

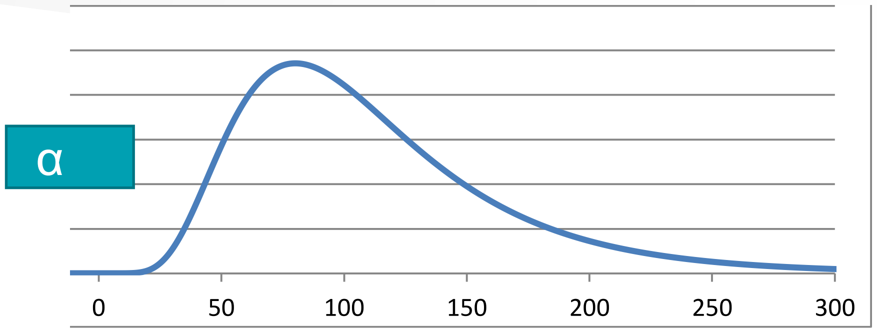
- Variance is range squared / 12:

$$\frac{(6H)^2}{12[4(1-\alpha)]^2} = \left[\frac{\sqrt{3}}{4-4\alpha}H\right]^2$$



Generalized Sizing Risk – Lognormal

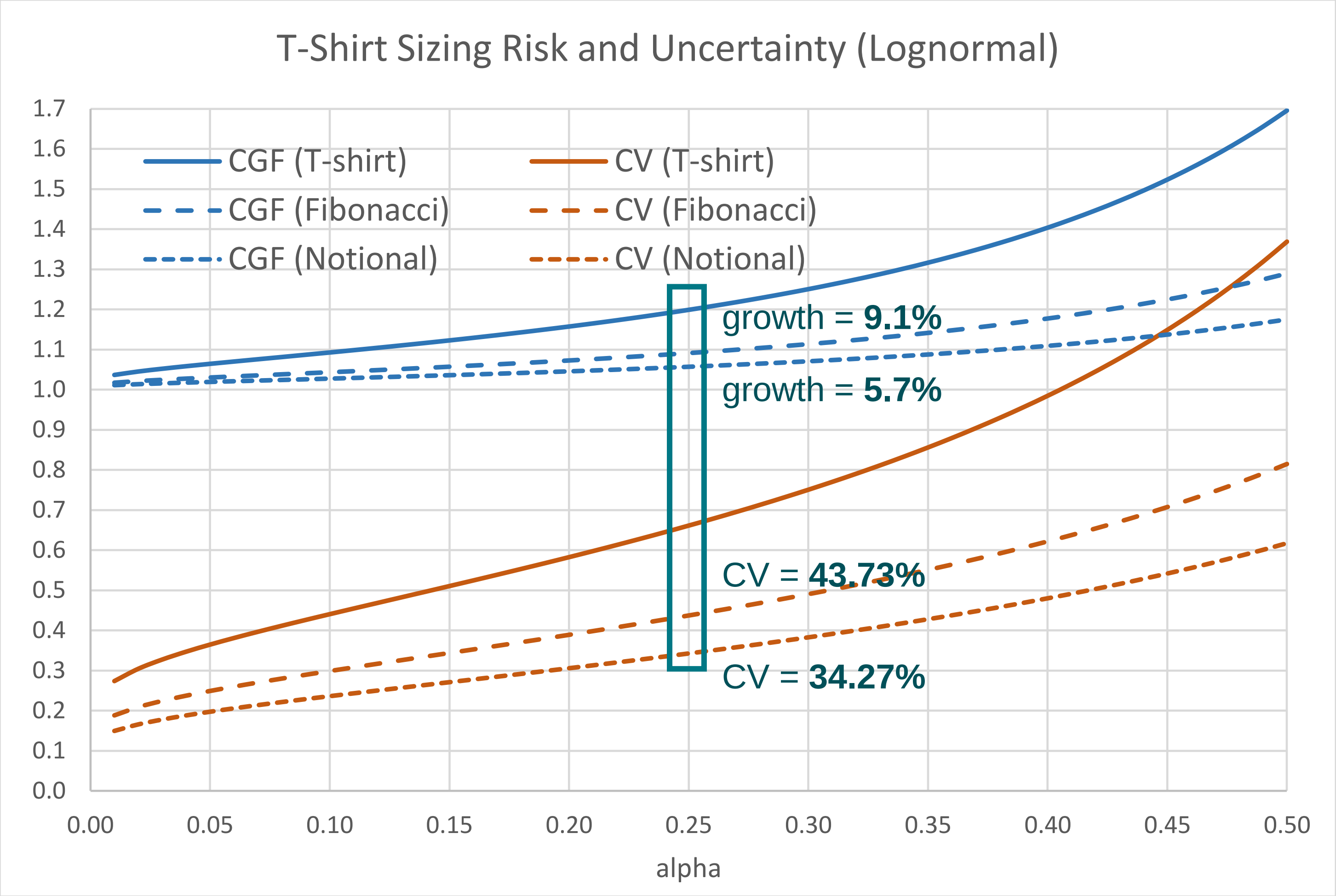
Refinement #4: Generalized Ratio



- Assume a Lognormal distribution:
  - Median = *H* hours, with a probability of 1-α between *H*/*R* and *RH*
  - Right tail > *RH* hours, with a probability of α/2
  - Left tail < *H*/*R* hours, with a probability of α/2
- Confidence interval of related normal is: $(lnH - lnR, lnH, lnH + lnR)$
  - So that $\Phi^{-1}\left(1 - {}^{\alpha}/_{2}\right) = \frac{lnR}{\sigma}$ $\sigma = \frac{lnR}{\Phi^{-1}\left(1 - {}^{\alpha}/_{2}\right)} = \frac{1}{log_R e^{\Phi^{-1}\left(1-{}^{\alpha}/_{2}\right)}}$
- Mean of the lognormal is: $e^{\mu+\frac{\sigma^2}{2}}$
  - With a CGF of $e^{\frac{\sigma^2}{2}} = \sqrt{1 + CV^2}$ $CV = \sqrt{e^{\sigma^2} - 1}$

Generalized Risk – Lognormal (Illustrated)

- Common factors shown for T-shirt sizing (2.000), Fibonacci (1.618), and Notional (1.467)



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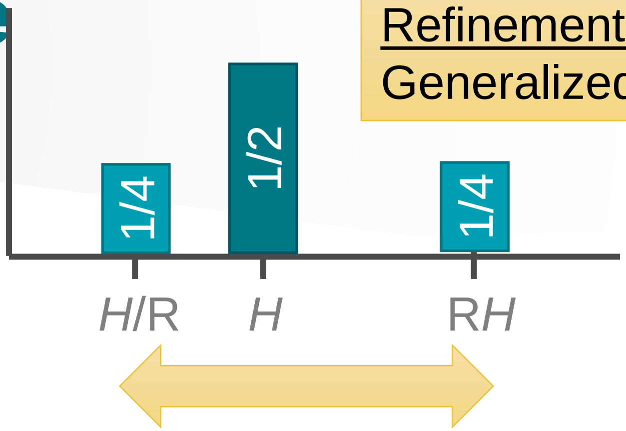
Generalized Sizing Risk – Discrete



Refinement #4: Generalized Ratio

- Assume a Discrete distribution:
  - Most Likely = H hours, with a probability of 1/2
  - Max = RH hours, with a probability of 1/4
  - Min = H/R hours, with a probability of 1/4
- Mean is expected value:

$$\sum_i x_i p_i = \left({}^1\!/_4\right)\left({}^H\!/_R\right) + \left({}^1\!/_2\right)(H) + \left({}^1\!/_4\right)(RH) = \frac{1}{R}\left(\frac{R+1}{2}\right)^2 H = \left[1 + \frac{1}{R}\left(\frac{R-1}{2}\right)^2\right] H$$

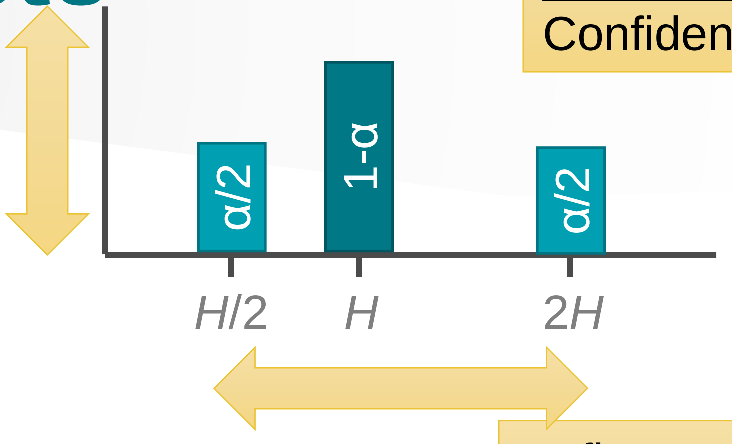
- Variance is expected value of square less square of expected value:

$$\sum_i {x_i}^2 p_i - \left[\sum_i x_i p_i\right]^2 = \frac{1}{4}\left(\frac{H}{R}\right)^2 + \frac{1}{2}H^2 + \frac{1}{4}(HR)^2 - \frac{1}{R^2}\left(\frac{R+1}{2}\right)^4 H^2 =$$

$$\left[\frac{3R^4 - 4R^3 + 2R^2 - 4R + 3}{(4R)^2}\right] H^2 = \left[\frac{R-1}{4R}\sqrt{3R^2 + 2R + 3}\right]^2 H^2$$

$$CV = \frac{R-1}{(R+1)^2}\sqrt{3R^2 + 2R + 3}$$

Generalized Sizing Risk – Discrete



Refinement #2: Confidence

Refinement #4: Generalized Ratio

- Assume a Discrete distribution:
  - Most Likely = H hours, with a probability of 1-α
  - Max = RH hours, with a probability of α/2
  - Min = H/R hours, with a probability of α/2
- Mean is expected value:

$$\sum_i x_i p_i = \left({}^{\alpha}/_{2}\right)\left({}^{H}/_{R}\right) + (1-\alpha)H + \left({}^{\alpha}/_{2}\right)(RH) = \frac{\alpha + 2(1-\alpha)R + \alpha R^2}{2R} H = \left[1 + \alpha \frac{(R-1)^2}{2R}\right] H$$

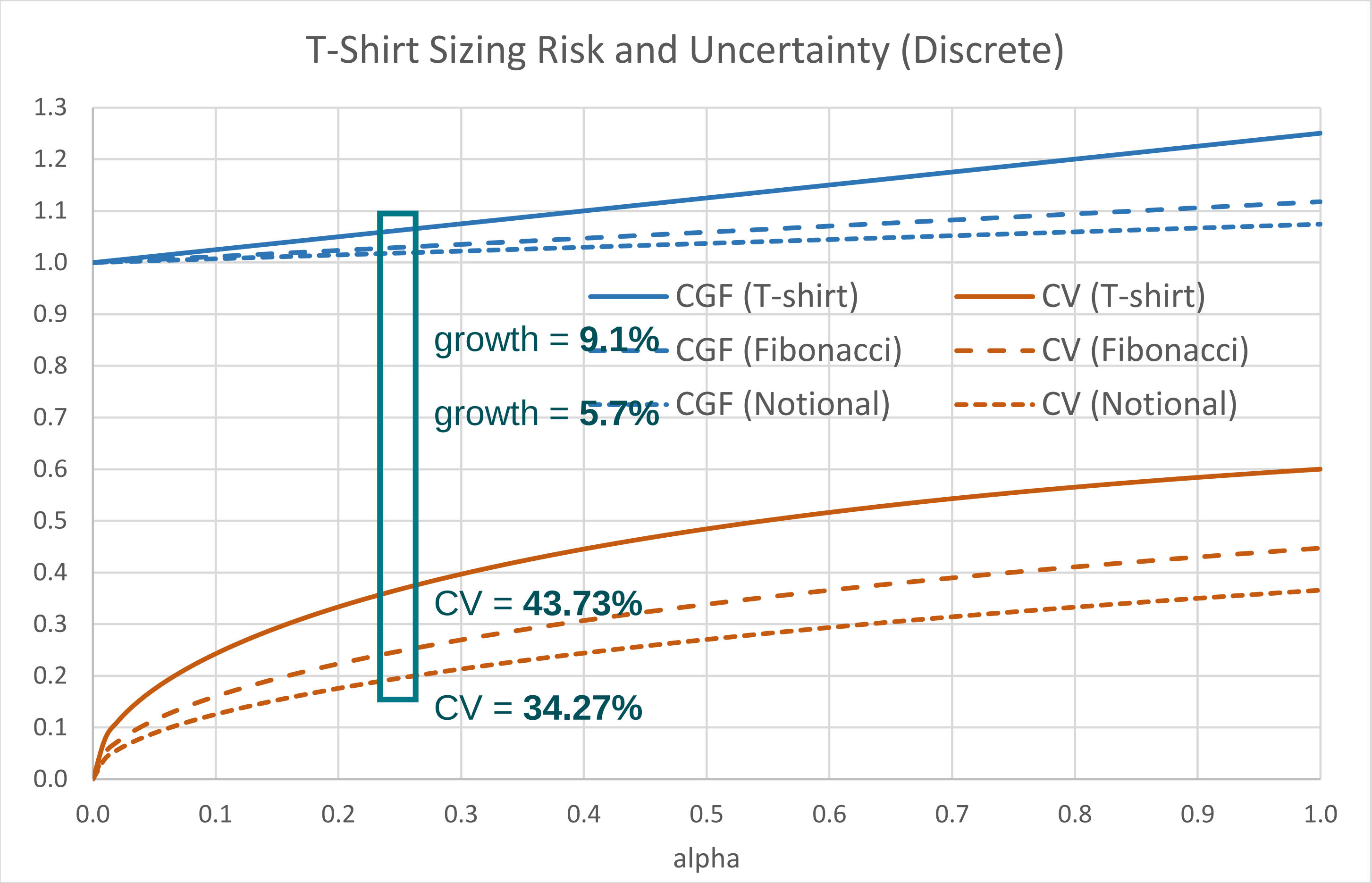
- Variance is expected value of square less square of expected value:

$$\sum_i x_i{}^2 p_i - \left[\sum_i x_i p_i\right]^2 = \frac{\alpha}{2}\left(\frac{H}{R}\right)^2 + (1-\alpha)H^2 + \frac{\alpha}{2}(HR)^2 - \left(\frac{\alpha - 2(1-\alpha)R + \alpha R^2}{2R}\right)^2 H^2 =$$

$$\left[\left(\frac{R-1}{2R}\right)\sqrt{\alpha[(2-\alpha)R^2 + 2\alpha R + (2-\alpha)]}\right]^2 H^2 \qquad CV = \frac{R-1}{2R + \alpha(R-1)^2}\sqrt{\alpha[(2-\alpha)R^2 + 2\alpha R + (2-\alpha)]}$$

Generalized Risk – Discrete (Illustrated)

- Common factors shown for T-shirt sizing (2.000), Fibonacci (1.618), and Notional (1.467)



Risk and Uncertainty by Confidence

- For confidence (1-α), we can express CGF and CV as a function of α
  - Generally, we would assume α < 0.50 (i.e., no worse than coin flip)

	Growth %	CV	Growth % (α = 0.25)	CV (α = 0.25)
Discrete (Generalized)	$\frac{\alpha}{4}$	$\frac{\sqrt{10\alpha - \alpha^2}}{4 + \alpha}$	6.2%	36.74%
Lognormal	$\sqrt{1 + CV^2} - 1$	$\sqrt{e^{\sigma^2} - 1}$	19.9%	66.16%
Uniform (Proportional)	$\frac{1}{4 - 4\alpha}$	$\frac{\sqrt{3}}{5 - 4\alpha}$	33.3%	43.30%
Uniform (Equal)	$\frac{1}{4}$	$\frac{\sqrt{3}}{5 - 5\alpha}$	25.0%	46.19%
Triangular (Proportional)	$\frac{1}{6 - 6\sqrt{\alpha}}$	$\frac{\sqrt{\frac{7 - 4\sqrt{\alpha}}{2}}}{7 - 6\sqrt{\alpha}}$	33.3%	39.53%

$$\sigma = \frac{1}{log_2 e^{\Phi^{-1}(1 - \alpha/2)}}$$

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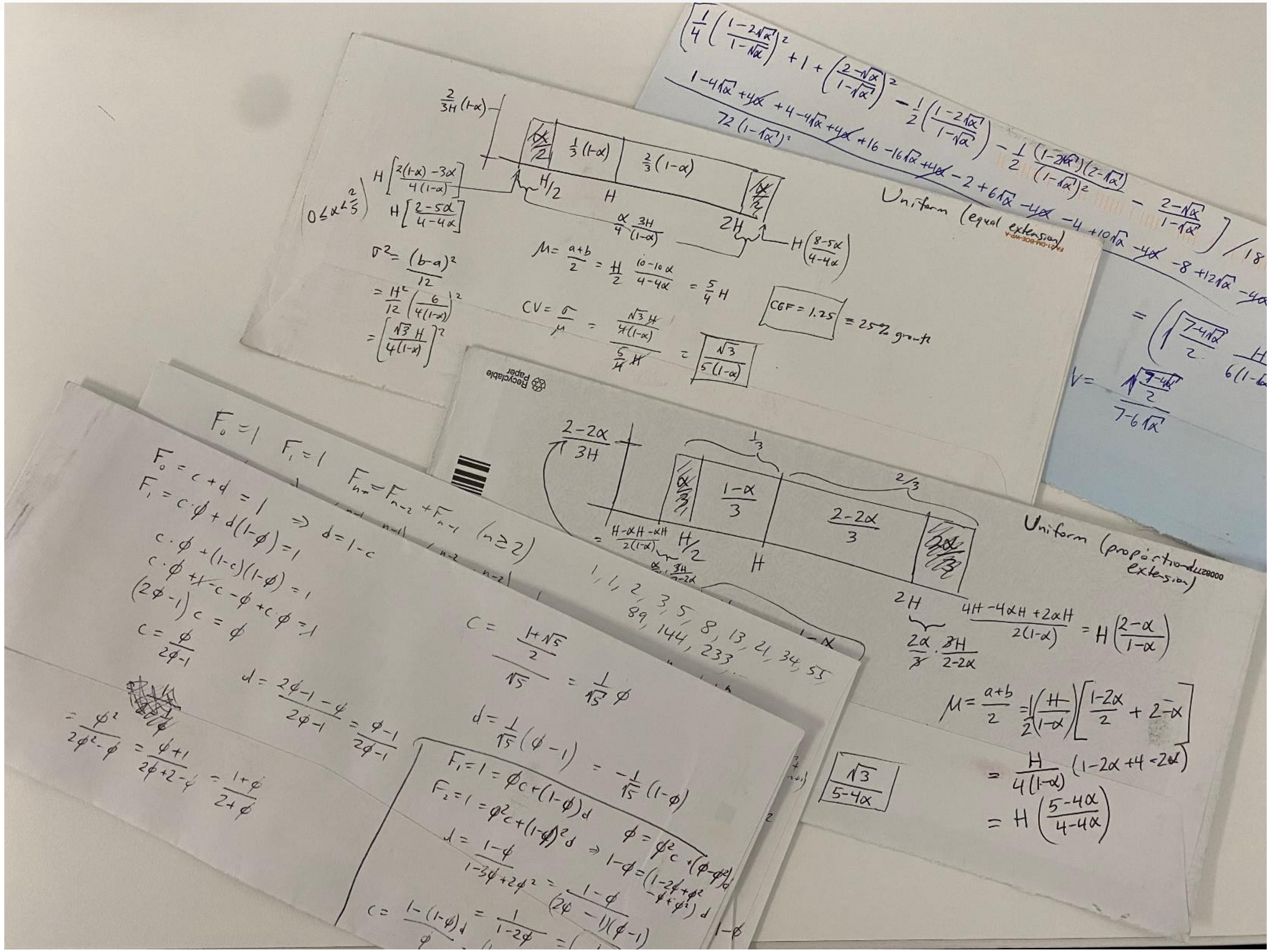
Summary (R = 2.0)

- The bottom line is that significant risk and uncertainty are inherent in these self-similar sizing scales *even if we are off by no more than one size in either direction*

	Confidence	Growth %	CV
Discrete	$\alpha = 0.50$	12.5%	48.43%
Uniform	$\alpha = 0.00$	25.0%	34.64%
Triangular	$\alpha = 0.00$	16.7%	26.73%
Discrete	$\alpha = 0.25$	6.2%	36.74%
Lognormal	$\alpha = 0.25$	19.9%	66.16%
Uniform (Proportional)	$\alpha = 0.25$	33.3%	43.30%
Uniform (Equal)	$\alpha = 0.25$	25.0%	46.19%
Triangular (Proportional)	$\alpha = 0.25$	33.3%	39.53%

Coda – The Proverbial Cocktail Napkin(s)



Example #1

Ghost in the Shell (2017)

Example #2

Paddington 2 (2018)

Example #3

The Homesman (2014)

Example #4

Ramona and Beezus (2010)

Example #5

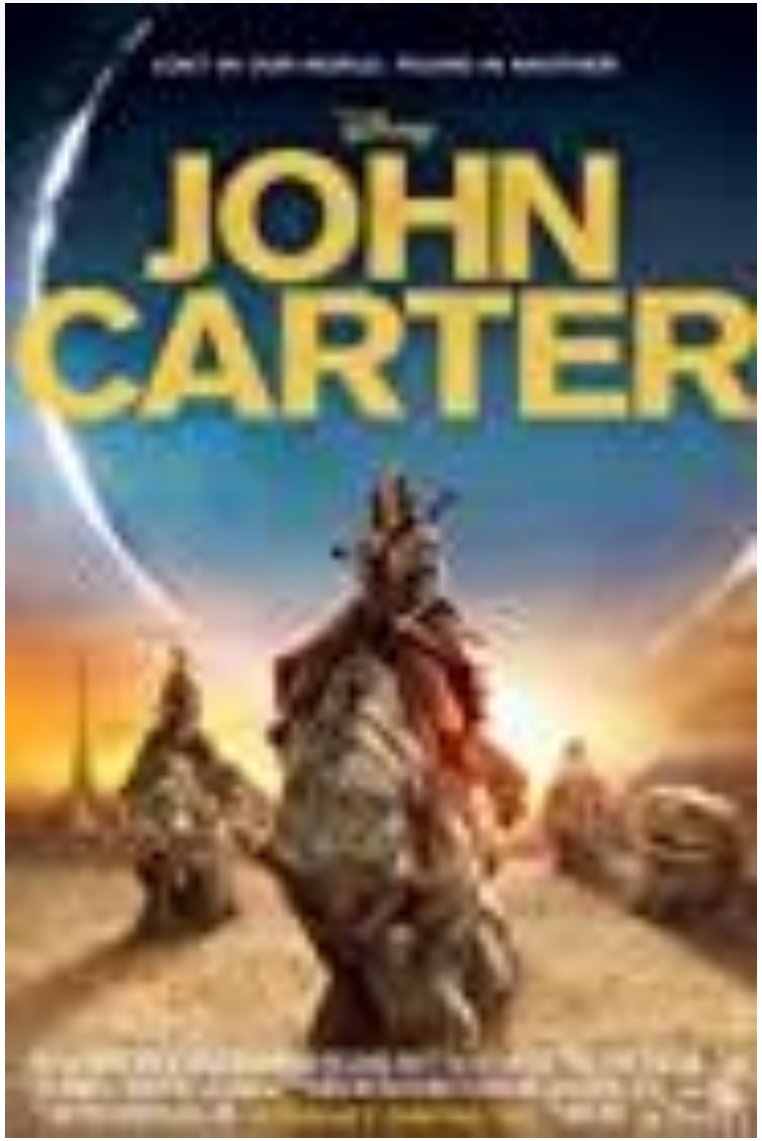
Rules Don’t Apply (2016)

Example #6

The Boss (2016)

Example #7

John Carter (2012)



Example #8

Me and Earl and the Dying Girl (2015)

Example #9

TRON: Legacy (2010)

Example #10

Toy Story 3 (2010)

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